Parametric Relations for Ordinary and Confluent Turbulent Boundary-Layer Flows

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Theme

THIS paper presents the results of an investigation of parameters associated with the confluent boundary layer shown schematically in Fig. 1. It is shown that with a suitable choice of parameters, velocity profiles in both jet and wake layers become similar, with the same similarity function obtained for different pressure gradients. Moreover, local dynamic similarity allows suitable functional representation of shear stress at the loci of maximum and minimum velocity. Measured eddy viscosity distributions for the confluent boundary layer is presented for one velocity ratio at slot exit. An analytical solution for the conventional flat plate turbulent boundary layer is shown for comparison purposes.

Contents

Experimental data were obtained in a wall-jet facility at Lockheed-Georgia Company. This equipment is described in detail by Goradia.[1] Ratios of slot velocity to freestream velocity ranged from 1 to 2 and pressure distribution was obtained by diffuser action. Only low flow velocities were considered to insure incompressibility.

Figures 2 and 3 respectively show similar velocity profiles in the jet and wake layers. The experimental data pertain to

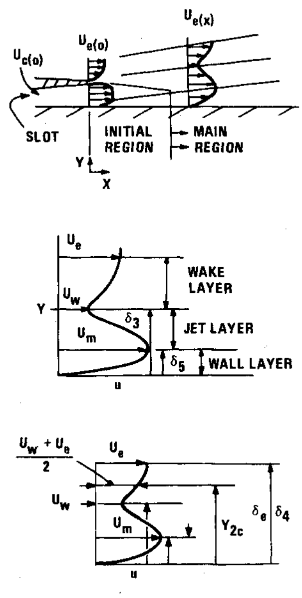

Fig. 1 Geometry.

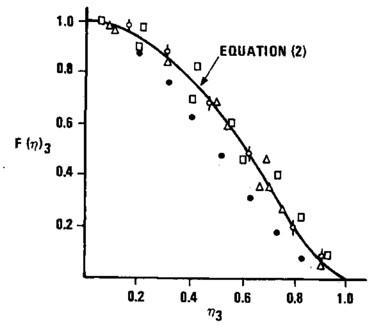

Fig. 2 Similarity of jet layer velocity profiles in the main region.

FOR ALL X'S IN THE JET LAYER

SYMBOL	INITIAL VELOCITY RATIO, $\frac{U_{c(o)}}{U_{e(o)}}$	PRESSURE GRADIENT
△	1.12	HIGH ADVERSE
□	1.50	ZERO
ϕ	1.67	MILD ADVERSE
•	ALBERTSON'S[2] DATA FOR TURBULENT MIXING BETWEEN TWO PARALLEL STREAMS	

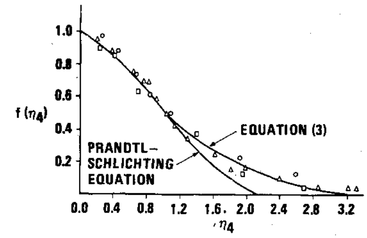

Fig. 3 Similarity of wake layer velocity profiles in the main region.

SYMBOL	$\frac{U_{c(o)}}{U_{e(o)}}$	PRESSURE GRADIENT
△	1.12	HIGH ADVERSE
□	1.50	ZERO
○	1.67	MILD ADVERSE

FOR ALL X'S IN THE WAKE LAYER

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Index categories: Boundary Layer and Convective Heat Transfer—Turbulent; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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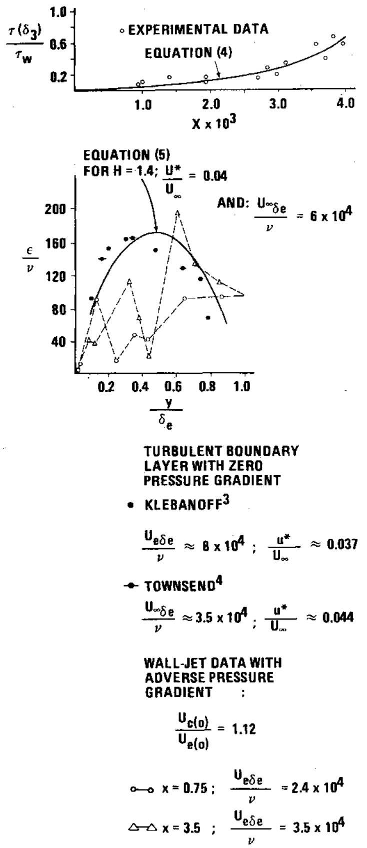

Fig. 4 Shear stress at minimum velocity location and eddy viscosity distribution.

different pressure gradients, x locations, and velocity ratios. Equations (1) and (2) are the least square polynomial curve fits for the similarity functions for jet and wake layers.

$$f(\eta_3) = 1.002 - 0.164(\eta_3) - 1.967(\eta_3)^2 + 1.338(\eta_3)^3 - 0.209(\eta_3)^4 \qquad (1)$$

$$f(\eta_4) = 1.0194 - 0.450(\eta_4) - 0.203(\eta_4)^2 + 0.1543(\eta_4)^3 - 0.024(\eta_4)^4 \qquad (2)$$

In addition, Fig. 2 shows Alberson's[2] free-jet data and Fig. 3 shows the well known Prandtl-Schlichting expression[4] for free turbulent wake flow.

The locus of minimum velocity U_w, i.e., the line $y = \delta_3(x)$, is common to both jet and wake layers, and hence the shear stress on this locus can be represented in the dimensionless form $\tau(\delta_3)/\tau_w = F(X)$ where

$$X = \left(\frac{\delta_3 - \delta_5}{\delta e}\right)\left(\frac{Y_{2c} - \delta_3}{\delta e}\right)\left(\frac{Um_{(x)} - Uw_{(x)}}{Ue_{(x)}}\right)\left(1 - \frac{Uw_{(x)}}{Ue_{(x)}}\right) \qquad (3)$$

Experimental data for various conditions are shown in Fig. 4 and a least square polynomial expression for the dimensionless shear on the locus of minimum velocity is given by

$$\tau_{(\delta 3)}/\tau_w = 0.465(10)^5(X)^2 - 0.0438(10)^3(X) \qquad (4)$$

For the conventional flat plate turbulent boundary layer, the theoretical expression for eddy viscosity based on Prandtl's momentum equation and power law velocity profile is

$$\frac{\varepsilon}{u^* \delta e} = \frac{u^*}{U_\infty} \cdot \frac{2}{(H-1)}\left[1 - \left(\frac{y}{\delta e}\right)^H\right]\left(\frac{y}{\delta e}\right)^{(3-H)/2} \qquad (5)$$

where u^* = friction velocity $= (\tau_w/\rho)^{1/2}$ and U_∞ is freestream velocity. Figure 4 shows a plot of Eq. (5) together with experimental data of Klebanoff[3] and Townsend[4] for comparison. Figure 4 shows, in addition, some typical variations of eddy viscosity distributions measured in the present project for wall-jet flow in the presence of an adverse pressure gradient. It is seen that the maximum value of eddy viscosity ratio increases as the distance from slot exit increases, and that for x locations in the initial and main regions the eddy viscosity distribution has more than one maximum or minimum. These minima and maxima occur near the junctions of the various layers.

References

[1] Goradia, S. H., "Confluent Boundary Layer Flow Development with Arbitrary Pressure Distribution," Ph.D. thesis, 1971, Georgia Inst. of Technology, Atlanta, Ga.

[2] Alberson, M. L., Dai, Y. B., Jensen, R. A., and Rouse, H., "Diffusion of Submerged Jets," *Proceedings of the American Society of Civil Engineers*, Vol. 74, 1948, p. 1751.

[3] Klebanoff, P. S. and Diehl, Z. W., "Some Features of Artificially Thickened Fully Developed Turbulent Boundary Layer with Zero Pressure Gradient," Rept. 1110, 1952, NACA.

[4] Townsend, A. A., *The Structure of Turbulent Shear Flow*, Cambridge University Press, New York, 1956.